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# Form as Diagram of Forces

## The Equiangular Spiral in the Work of Pier Luigi Nervi

The relationships between form, performance, and construction are uniquely demonstrated in the long-span works of Pier Luigi Nervi. The balance of these forces led in Nervi's case to a series of works that take the form of the equiangular spiral, a bizarre geometrical phenomenon that appears regularly in the natural world. The affinities between Nervi's work and the spiral's natural occurrences can be explained via D'Arcy Thompson's *On Growth and Form*, suggesting that this familiar book on biological morphology be seen by architects in a new and more thorough light.

> *The form, then, of any portion of matter, whether it be living or dead, and the changes of form that are apparent in its growth, may in all cases alike be described as due to the action of force. In short, the form of an object is a "diagram of forces."*[1]
>
> — D'Arcy Wentworth Thompson, *On Growth and Form,* Introductory

D'Arcy Thompson's *On Growth and Form* contains within its literate prose the profound suggestion of a morphology based on empiricism and mathematics — a powerful challenge to our established architectural traditions of style and composition. Thompson, of course, had no such motivations in writing, and, despite his occasional use of architectural (or more commonly structural) metaphors, it seems unlikely that his work's translation into the field of architecture would have held more than a passing interest for him. Yet his masterwork has an unavoidable relationship to the work of the human designer. His examinations of the formation and performance of matter in biology contain lessons and examples with inevitable consequences for architecture, in which form is also at least in part determined by both assembly and function. In architecture, the conscious intelligence of the designer takes the place of the latent intelligence accumulated by natural selection; perhaps more importantly, the operations of organic and constructive processes are likewise related but distinctly conceived. The organism grows by gradual accumulation over its life, the architectural structure by relatively instantaneous assembly at its inception. Relationships between organic morphology and architectural form, therefore, can be expected to show dialogical affinities, rather than determinant linkages.

This article explores the action of such affinities in the work of Italian architect, engineer and *constructeur* Pier Luigi Nervi (1892–1979). In Nervi's case, these affinities emerged with striking clarity in a set of long-span works, demonstrating the potential richness of an empirical design process and hinting at a limited though important correspondence between the world of the organic and that of the constructed. In these works, the equiangular spiral emerged as a result of Nervi's ability to balance forces of construction and performance. Nervi's roof forms suggest that the strongest link between the organic and the constructed exists not at the level of visual or formal representation, but rather in the deeper structures of geometry and mathematical patterning. Although metaphor and the imitation of striking natural forms may be architecturally tempting, Nervi's work suggests an "organic" that lies in the relationships among constructive and structural processes and the deployment of material to enable these. This theme, which also permeates Thompson's work, connects humankind's constructions to the natural world through the notion of design as an arrangement of resources for benefit, not design as the construction of spatial or graphic experience. The resulting forms, whether they are natural or human made, display patterns and formations that we recognize as beautiful or engaging not simply because of their visual proportions but rather because of our innate recognition of their mathematical efficiency, their logic, and their patterning.

A comparison between these two realms reveals the mechanisms of this organic beauty in algorithmic geometry and suggests both the potential for and the limits of the conceptual link between organic and structural morphogenesis. In other words, rather than formal or visual connotations, a Thompsonian organic will involve parallels between the natural and the man made that lie in the processes of growth, construction, and function. More particularly, these affinities will lie in the persistence of simple geometries and patterns that are, in Steven Jay Gould's words, "optimal representations" of the physical forces at work on a given object, be it architectural or natural.[2]

### Mechanism and Teleology— Thompson's View on Growth and Performance

Sir D'Arcy Wentworth Thompson's *On Growth and Form* has achieved a cultlike following among theorists and practitioners of architecture. It is an extraordinarily compelling work of biological explanation, written by a scholar whose background effectively bridged quite neatly the two cultures of science and the humanities.[3] Its fundamental prem-



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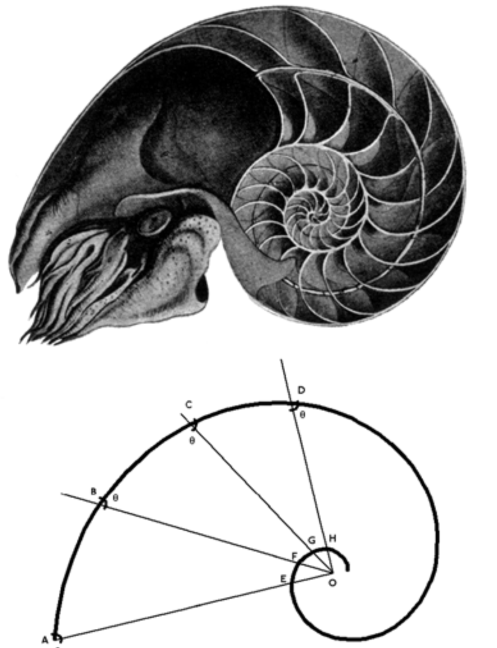
ise — that organic structures conform to both laws of physics and rules of efficiency in their growth patterns — is easily lost amidst Thompson's ecstatic descriptions of natural phenomena. Yet it remains readable if erudite, and it has thus impressed and inspired a range of architects and engineers even as its hypotheses have been modified by more recent developments in evolutionary biology.[4]

The place of *On Growth and Form* in the history of biology demands clarification, primarily for its critique of Darwin's reliance on function to explain evolutionary formation.[5] The traditional teleological "argument from design" was arguably in Darwin's case only slightly modified to substitute the iterative logic of evolution for a divine plan. The overall thrust of Thompson's argument, all too often lost in architectural references, was an attempt to explain organic formation through the processes of motive or efficient causation. Whereas Darwin's evolutionary theory was primarily concerned with outcomes, Thompson's was primarily concerned with means: no matter how advantageous a hypothetical adaptation might be, its formation must be physically achievable given the organism's milieu. The resultant "weaving together" of the "warp and woof . . . of mechanism and teleology" took its inspiration in part from Aristotle's *The Parts of Animals*.[6] As Thompson pointed out, Aristotle's relationship between final and motive causation was best described with an architectural metaphor: "the house is there that men may live in it; but it is also there because the builders have laid one stone upon another."[7] Form may thus be seen as the resultant of these two sets of forces: construction and performance. In the case of Nervi, the description of these forces as interwoven will be particularly apt, as his process suggests a fluid integration of the necessary function, with constructive and structural methods.

Although portions of *On Growth and Form* dealt with issues of performance, the dominant theme of the book was that of process — the formation or assembly of organic structures. Throughout




1. The equiangular spiral and its geometric properties, along with its most acclaimed occurrence in nature, the nautilus shell. (Diagram by the author, image from Theodore Andrea Cook, M.A., F.S.A., *The Curves of Life* (rep. New York: Dover, 1979).)

numerous examples, Thompson demonstrated that process invariably plays a role in the determination of organic form and that efficiency in formation (motive cause) is as determinant a vector in morphology as efficiency is in performance (final cause). Underlying nearly all formative processes, Thompson found mathematical principles or algorithms that organize and streamline growth processes and that reveal startling similarities in organic forms across and throughout species.

The equiangular, or logarithmic, spiral was Thompson's most striking example of an algorithm made manifest, for both its compelling geometry and its widespread occurrence in diverse natural phenomena. This spiral is the product of a simple geometric algorithm, consisting of the path traced by a moving point simultaneously sweeping around and accelerating away from a fixed center.[8] The resulting shape is at once self-similar — any segment of its curve is exactly proportional to any other — and constant in its local angular relationship to its central point.[9] The spirals possess "gnomonic" properties, such that any portion of the spiral, when removed or added, produces a resultant shape proportionally similar to the original. The spiral's most striking architectural property — its relationship to the Golden Section — results directly from these properties. Because of the spiral's self-similarity, a series of nesting shapes laid out with vertices at regular polar intervals along the path of an equiangular spiral will be similar to one another and to the overall composition; the addition of the gnomon will create a new figure proportionally similar to the old. In the case in which each additive shape is a square, the resulting figure will be a rectangle with proportions of 1:1.618034, the so-called Golden Ratio.[10] The spiral's self-similarity or "ratio repetition" also tends to produce proportions based on the Fibonacci series — the series of numbers in which each term is the sum of the previous two, and in which the ratios of successive terms approach 1.618034:

1 1 2 3 5 8 13 21 34 55 89 144 . . .

For Thompson, however, the spiral was most simply a signature of staged, exponential growth, or, rather, accumulation: the spiral exists in dead matter as it is not a result of constant growth but is rather a by-product, the result of an organism accreting or excreting matter over time.[11] This temporal aspect is a key element in the spiral's manifestation, as the activity of adding to a shell, or horn, is necessarily based on the previous assembly's form and scale. Such phenomena as shells, horns, or teeth all share this process of development, and Thompson therefore deduced that these spirals arise as the results of efficient algorithms, in that they represent the production of dead matter in constant relation and proportion to forms already produced. Put in Thompson's language, the result is growth in size without change of shape. In cases such as the nautilus shell, this can be simplified even further in that the shell's fabricational instructions can be stated simply as the result of constant accretion of the outer shell at a given, positive angle to the existing, inner shell. On reflection, it is in fact unlikely that such a process could create any form other than an equiangular spiral.[12]

2. Second generation of hangar at Orvieto, Pier Luigi Nervi, ca. 1938. (From Pier Luigi Nervi, *The Works of Pier Luigi Nervi* (New York: Praeger, 1957).)

This algorithmic rigor, evident throughout Thompson's examples, presents a powerful challenge to architectural form making. Through Aristotle's analogy one can see obvious parallels. Architecture is concerned with both motive and final causes; however, forces of commodity often overtake those of assembly and function. Our resources in terms of materials and constructional algorithms are not always determined by necessity, and thus the rigor that accompanies natural morphology is not always present in architectural situations. Nevertheless, in situations of extremity — long spans, tall structures, and so forth — processes of making and shapes derived from static models become more determinant, and the resulting forms more closely approach the ideal geometries dictated by pure physics. The following investigation of Nervi's work will demonstrate the confluence of final and motive causes, under circumstances of extreme constructional duress, into structures with precise parallels to the aforementioned organisms, elucidating an approach that relies on rigorous adherence to the efficiencies of constructional algorithms and structural principles.

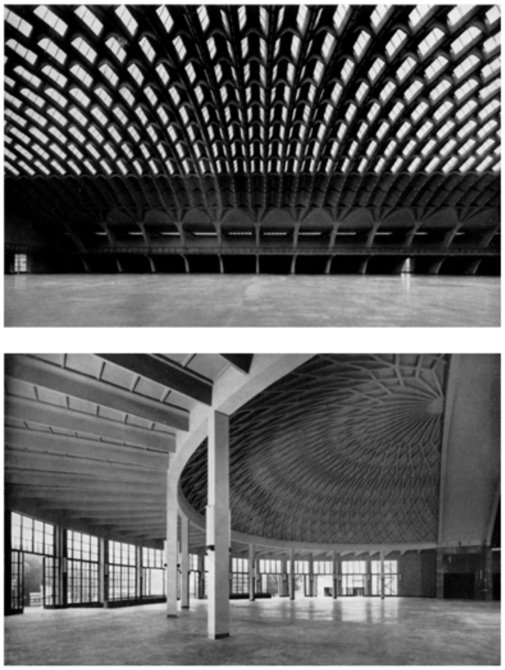
## Means and Motive: The Development of the Lamella Surface in Nervi's Roof Forms

Thompson's masterwork was read and quoted by a number of designers and critics in the late twentieth century, both for his authoritative notes on organic morphology and his remarkable aphorisms — visual and literary — regarding efficiency in the performance and construction of natural examples. The 1942 reprint, which abridged the original nine-hundred-page work, appeared regularly in architects' libraries, including those of Mies van der Rohe and Louis Kahn.[13] Perhaps the most intriguing example of its influence was Myron Goldsmith's 1953 thesis, "The Effects of Scale," which quoted Thompson on the exponential relationships between scale and efficiency as the basis for his own research on scale in engineering.[14] Kahn's introduction to Thompson was via Anne Tyng, whose work on the Jewish Community Center shows parallels to several "close packing" examples illustrated in *On Growth and Form*.[15] In the 1960s, Frei Otto made numerous direct references to Thompson's examples, finding in Thompson's descriptions of minimum a particularly rich source for the developing theories of tension and pneumatic structures.[16] Most famously, Thompson's demonstrations of geometrical packing and efficient surface structures were used (although rarely credited) by Buckminster Fuller in his tetrahedral and spherical geometries. More recently, the authority of Thompson's examples has been used by Kenneth Frampton to define the "biomorphic . . . clustered" forms of Alvar Aalto and Jorn Utzon against the rectilinear "modular" forms of Le Corbusier.[17]

Although these examples have proven both illuminating and challenging to traditional conceptions of architectural form, it is surprising that the full scope of Thompson's argument has not been examined in relation to the processes of assembly and performance that accompany building construction. Lost in the metaphorical relationships between the natural world and the built are the deeper connections between motive and final causation demonstrated in Thompson's examples. The import of *On Growth and Form* for architecture is not merely that there are correspondences among natural and man-made forms, it is that the reasons for these correspondences have to do with the balance between how and why a thing is made. Particularly considering the expressly architectural example given by Aristotle, it seems worthwhile to propose that D'Arcy Thompson's work be seen as not only a provocative catalogue of natural examples, but as a critical framework for analyzing forms — both natural and constructed — as precisely this balance between fabrication and function. Seen thusly, both biological and architectural examples should reveal a wealth of constructed or embodied intelligence. Underlying the logics of assembly and performance, we should not be surprised to find parallels, although not precise copies, of patterns and algorithms that ensure efficiency in the deployment of material as forms are generated through biological or constructive processes.

A number of architects and engineers come to

3. Turin exhibition hall, Pier Luigi Nervi. 1948. Views of Salon B, main hall, and apse. (From Pier Luigi Nervi, *The Works of Pier Luigi Nervi* (New York: Praeger, 1957).)



mind as potential test cases for such a Thompsonian analysis, one that would parse the design and construction histories of built examples into motive versus final causation. Pier Luigi Nervi is an ideal candidate, as his background included training as an engineer and apprentice work as a contractor, thus providing him with an intellectual background that is inclusive of both means and ends as determinant factors in building. There is no record of Nervi mentioning *On Growth and Form* in a public setting, and no such link seems to have been drawn yet between Thompson's thought and Nervi's constructions. In Nervi's consideration for integrating the means of construction, the performance of engineering, and the expression of architecture, however, there are strong parallels between the "weaving together of warp and woof" in the biological examples of Thompson and the careful balance between three "essential problems" in Nervi's work.[18]

Writing in 1963, Nervi defined his work to that date as an overall manifestation of "structural architecture," whose major points he summarized as follows:

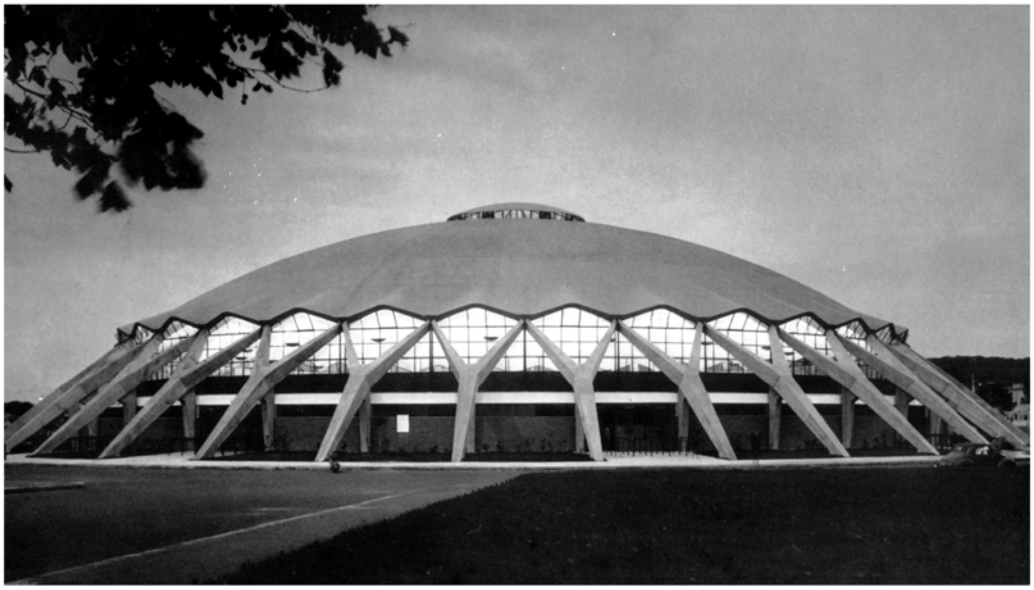
> I believe the essential conditions of structural architecture to be as follows:
>
> 1. It must give a convincing answer to a real and authentic static necessity and be determined by it.
> 2. A static constructive scheme should become visible and comprehensible inside and outside.
> 3. It must express frankly the material with which the structure is executed and find in the technological characteristics of the material itself the sources and ways, as well as the details of its architecture.[19]

This definition is a succinct expression, in architectural terms, of Thompson's "warp and woof" integrated into a singular, integrated approach. If Nervi's first and last points can be seen as analogous to final and motive causes, respectively, it is important to note the subjective nature of his second. Nervi's trifold training as engineer, architect, and constructor is very much in evidence, and his philosophy is not satisfied to merely arrive at the most efficient solution, although that is clearly part of the designer's charge. One must also strive for a communicative aspect, based in the clear revelation of the forces at work.[20] Thompson's notion that "form" may be analyzed as a "diagram of forces" is in Nervi's work thus reversed: "form" may also be synthesized from precisely such a static diagram as well, through the means and expressed in the visual language of construction. This process may be seen most clearly in Nervi's development of the "lamella" structural principle in a series of works executed from 1935 to 1960.

In 1935, Nervi received a commission for a series of aircraft hangars to be built at Orvieto for the Italian Air Force. The scheme proposed by Nervi was one of extreme simplicity and efficiency. To span the long distance required by the size of the aircraft, Nervi proposed a lamella shell structure, essentially a series of directionally opposed intersecting arches set diagonally to the rectilinear volume below. The resultant diamond pattern can be thought of as either a space-frame wrapped around a cylindrical volume or as a barrel vault with much of its dead weight removed, in either case combining the actions of latticed roof structures with an arched shape and allowing an extraordinary span/weight ratio. Because of the lamella pattern's inherent triangulation, each joist is capable of carrying both gravity and lateral loads, and their proximity and connections to one another allow loads to be distributed throughout the network, rather than carried by a single member. Deployed along curved surfaces, lamella joists offered the load-carrying abilities of a long-span arch with the light weight of thin-shell concrete, or, thought of another way, the possibility of long-span compression members with very short unbraced lengths.[21] Reflecting the state of construction and design technology at the time, the hangars were built of intersecting circular cross sections. Catenary sections would have been structurally ideal, but the consistent shape of the radiused roofs approached the proportions of the perfect shape while allowing each bay to be of uniform dimensions to ease construction — a notable compromise between forces of assembly and performance at the largest scale.

The first pair of these hangars were built of poured-in-place concrete, using removable formwork to obtain narrow arch profiles. Although the material savings from the structural solution were significant, Nervi recognized inherent flaws in this process. As the next phase was also put to competitive tender five years later, Nervi refined both the structural scheme and the constructional process. The ribs of the next set of hangars were prefabricated, saving costs in labor and formwork. Forming these ribs on the ground allowed more intricate casting, which meant that ribs not subject to large shear or bending forces could be fabricated as lat-

4. The Palazetto dello Sport, Rome, Rome. Pier Luigi Nervi, 1958. Exterior view showing forked columns.
(From Pier Luigi Nervi, *Buildings, Projects, Structures: 1953–1963* (New York: Praeger, 1963).)



ticework, lightening the structures' dead loads considerably. The second set of hangars proved dramatic savings in material and construction time before being destroyed by retreating forces in World War II. The development of two distinct fabricational methods reflects the evolutionary nature of Nervi's process, as the second iteration maintained the structurally efficient geometry of the original while implementing a more efficient construction method — a parallel to Thompson's version of natural selection, in which algorithms are "chosen" for their material economy.[22]

Nervi did not often explore the static and constructional efficiencies of the lamella shell system in subsequent projects, as his long-span work focused instead on corrugated or folded-plate elements. However, two ideas from these early hangars run throughout his later work, namely the use of precast beam elements and "coffered" roof spans where structural depth is maximized along lines of isostatic force. A third consistent element in Nervi's work, the use of "ferro-cement" formwork left permanently in place, addressed an intense shortage of timber in postwar Italy and the desire to maintain control over the internal appearance of the expressed structure. The problem of formwork was fundamental to the nature of concrete construction for Nervi, as previous designers had been limited by a reliance on timber or steel formwork to contain concrete while it cured. This led, in Nervi's mind, to an inarticulate expression, as it necessarily adopted the forms and markings of timber construction and not those intrinsic to the concrete itself. Ferro-cement formwork was made by injecting rich concrete mortar into molded wire mesh, creating a very thin, durable panel that took advantage of the steel mesh's capacity in bending. More importantly, the panels could be any shape desired, particularly curvilinear geometries reflective of concrete's poured nature and the gently arcing lines of static force in slabs and arches.[23] These shapes could be hand molded with accuracy by forming the wire mesh over preset jigs, allowing multiple units to be made reliably and quickly. The use of ferro-cement eliminated the need for timber and in fact took advantage of the region's one major economic resource, an abundance of inexpensive labor.[24]

In 1948, Nervi competed for and won the contract for a new exhibition hall in Turin. With fewer than eight months for construction, and a program requiring spans of more than 100 meters, Nervi was again faced with the need to produce a scheme of constructional efficiency. He therefore proposed a series of poured-in-place concrete buttresses supporting a corrugated shell of precast elements. The roof elements, made of ferro-cement, repeated a structurally efficient folded-plate module hundreds of times over the surface of the roof. A series of intermediate diaphragms gave each unit its own integral stability while providing adequate contact area to ensure overall monolithic performance. The result was a finely grained roof structure, elegantly collected at its base by a series of fan vaults that added to the hall a sense not only of scale, but also of process — a visualization of forces both constructive and static.[25]

Nervi produced a number of long-span halls based on the Turin model; however, a relatively small apsoidal hall at the end of the main exhibition space is of particular interest to the study of Thompson's morphogenetic theories. This apse had been conceived as a response to the oddly shaped site, a semicircular space that extended the main exhibition area at a smaller scale. Because of the static geometry of the main hall, there was no way to carry the logic of the corrugations into this element, and instead Nervi devised an ingenious system of radially arrayed coffers to transmit the weight of the dome to a colonnade beneath. These coffers were formed of diamond-shaped *ferro-cemento* pans arrayed in an efficient structural pattern, following lines of gravitational and torsional stress down to the colonnade.[26] The final effect is visually striking, but the logic of the construction and structural performance of this system is perhaps more compelling. While forming an efficient, monolithic structural system of in situ concrete, the *ferro-cemento* pans ensured a consistent appearance based on the quality achievable by precasting, and their mass production allowed dramatic savings in cost and schedule. It is apparent from the final appearance of the dome, however, that Nervi was not yet fully fluent in the spherical geometry of the structure, as the proportions of the pans vary widely toward the top of the dome. This is not merely an aesthetic shortcoming; it meant each family of pans required a separate set of jigs for their formation.

In subsequent designs, Nervi improved on this method with dramatic results. If the aircraft hangars of the 1940s were pure lamella roofs, essentially wrapping a diamond pattern around a cylinder, later

5. The Palazetto dello Sport, Rome. Pier Luigi Nervi, 1958. Interior view showing pattern of concrete ribs. (From Pier Luigi Nervi, *Buildings, Projects, Structures: 1953–1963* (New York: Praeger, 1963).)

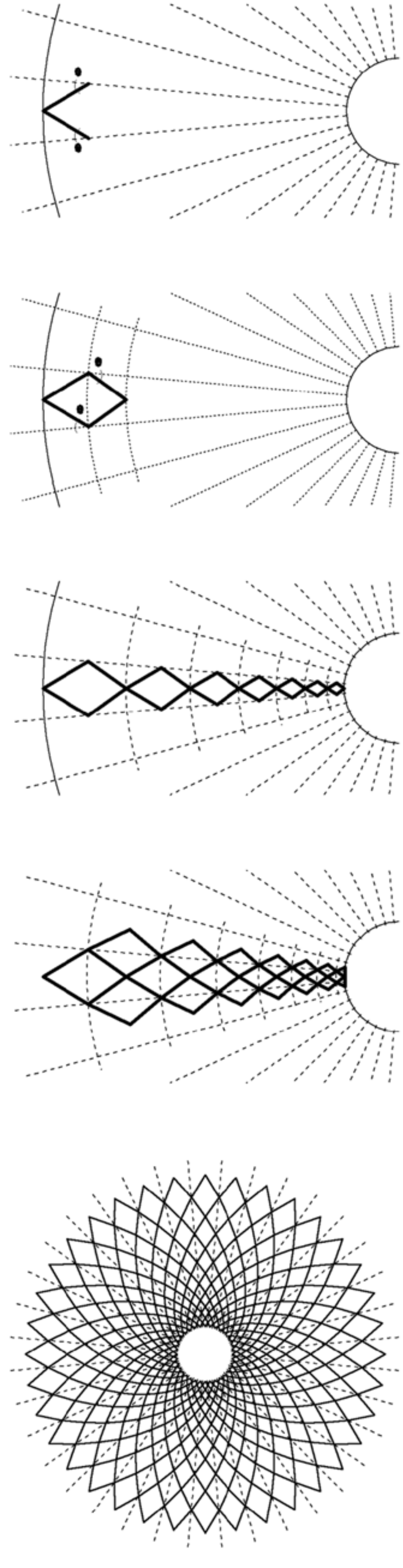
projects represented a rotational interpretation of the lamella principle. Instead of a series of diagonal arches on a rectilinear plan, the roof structures of these projects can be seen as diagonal arches based on a polar plan, in other words, with regular angular relationships and proportions based on their radial distance from the center. The resultant shapes provided an efficient distribution of gravity loads along the surface of the roof, while at the same time providing a geometrically based resistance to lateral and torsional forces, avoiding the inefficiency of the Turin apse through regular triangulation.

The best known of Nervi's rotated lamella domes is the small Olympic arena built for the 1960 games in Rome. Although the larger indoor space, the Palazzo dello Sport, consisted of a prefabricated trough system, analogous to the first Turin exhibition hall structure, the Palazetto dello Sport represented Nervi's purest realization of the radial pan lamella system. From the outside, the overall structural action of the dome is clearly expressed. The dome's joists are gathered at the structure's perimeter into a series of fan vaults, which are in turn supported by thirty-six Y-shaped buttresses that take gravity and thrust loads to ground foundations. The exterior expression itself, combined with the careful placement and junction of the glass curtain wall enclosure, provides a clearly conceived and lucidly expressed resolution of the forces involved, using job-cast members whose forms are manifestations of the dome's static vectors. Between the fan vaults, where enclosure is required but where the dome's loads are not being carried, the concrete shell is pulled upward into an eyebrow, admitting daylight to the concourse area while indicating that the concrete here is carrying no load from the dome above.[27]

The elegance of the Palazetto's exterior is, however, far surpassed by its striking interior, whose genesis merits careful examination. In "bending" the lamella principle into a radial form, Nervi faced the geometrical problem evident in his earlier work, in that the 1,620 diamond modules of the roof necessarily changed in scale based on their distance from the center. In his Turin experiment, the results had been inconsistent, necessitating modules varying in not only size but also in proportion to fill the domical surface. At the Palazetto, this problem was solved via a simple geometric algorithm.[28]

Exactly how Nervi's proportioning system developed is not known, but it is enlightening to reconstruct one possible scenario. This elucidation of Nervi's process suggests an affinity to Thompson's reliance on motive causation to explain form, as it stems from an economic requirement to minimize variation in geometry throughout the system. Such consistency relied on a repetition not only of the molds used to bend the wire cages on which the ferro-cement was placed, but also of the jigs used to make the molds, as the angles subtended by each edge could be kept consistent throughout the project.[29] In addition, reinforcement placed in the resulting troughs formed by the edges of the pans could be designed to meet at the same angle throughout the project, allowing mass production of steel connections between embedded reinforcing bars.

The essential problem was thus to ensure a full tiling of the domed surface with similarly shaped and consistently proportioned precast elements, ensuring that each one would subtend consistent angles throughout the dome. To do this, one can start with an external perimeter and an assumption about the inscribed angle of each pan — in the case of the Palazetto, 3-1/3 degrees. Next, the angle of incidence for each joist is figured, based on the maximum size achievable for each pan and on allowable stresses in each joist.[30] The resulting intersection of joist and circumference determines the setting-out point of the next half-diamond pair of joists, essentially reflected through the newly derived circle. This process is then continued until a dimension arbitrarily near the center of the dome is reached, based on a balance between the weight of the dome's cap and the decreasing size of the pans beyond the point of fabricational convenience. A second series of pans based on the intermediate



6. Derivation of the lamella dome pattern. The repeated use of proportionally sized pans creates, inevitably, a series of interlocked equiangular spirals. (Diagram by the author.)

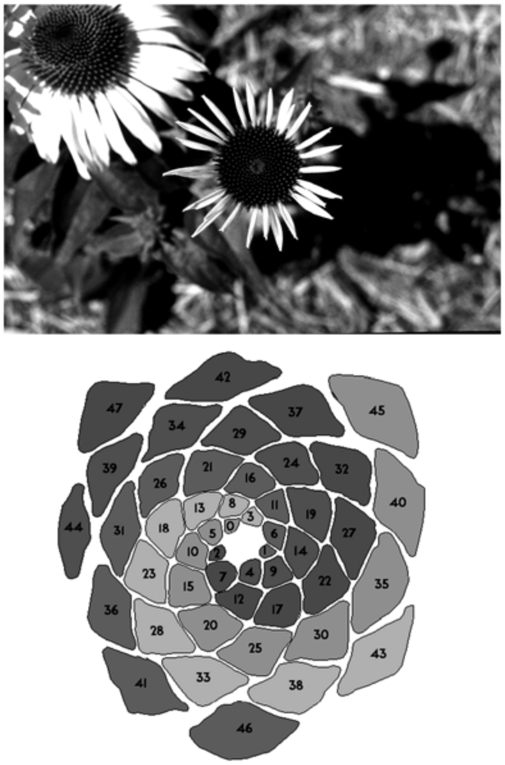
dimensions of the first series completes the fabric of a typical radial bay. Each joist is created by the downstand edges of adjacent pans, which form a void into which reinforcement and concrete are placed.

When this typical bay is then rotated about the center to form a complete dome, the self-similarity of each joist to one another and the consistency of the joists' angular relationships give rise to a series of interwoven logarithmic spirals. That the spirals should be manifest in such an endeavor should not be at all surprising, for the processes of assembly involved here are similar to those undertaken by the nautilus, or indeed by any of the organisms cited by Thompson as constructors of equiangular shells. In each case, an element composed of dead material — whether secreted or formed — is produced adjacent to a geometrically similar, gnomonic element with only a regulated change in scale to differentiate one from another. The role of time and sequence is important to the assembly of each — although in the nautilus the process occurs from the inside out, whereas at the Palazetto the construction proceeded from the perimeter inward. Through self-similarity, gnomonic geometry, and carefully ordered process, the pans in the Palazetto and the shell elements of nautilus both manifest the physical documentation of an ultra-efficient constructional algorithm — the performance of a repetitive fabricational process in series, with a constantly changing scale coefficient.

Intriguingly, the roof of the Palazetto bears a resemblance to another instance of the spirals in the natural world, namely the interweaving of florets in leaf formation. This process, known as spiral phyllotaxis, is covered in a chapter of Thompson's book that is usually abridged, both for its lack of clarity and Thompson's frustration that science had not yet fully explained the process.[31] The mathematics that govern the growth and patterning of leaves and florets produce curves and patterns similar to those of the nautilus, albeit overlapped and internally interactive in complex ways. Like the Palazetto and the nautilus, phyllotaxis is an immediately visible example of the role played by geometry in both assembly and structure, in that the spatial organization of its constituent elements is a result of efficient algorithmic programming, here in multiple dimensions.

Most immediately analogous to Nervi's system are the patterns of seed formation in the capitula of sunflowers.[32] As Thompson explains, these florets — or "primordia" — are produced at regular intervals at the capitular edges, gradually migrating toward the center while constantly increasing in size. That they maintain their rough shapes while so doing influences the geometry of the overall scheme, producing vibrant spiral patterns; however, there is also an algorithmic process to their movement and development that regulates the arrangement of the primordia into these carefully ordered patterns. Known as "parastichial spirals," these patterns of florets maintain recognizable allegiances to the Fibonacci series. In general, the number of spirals counted clockwise and counterclockwise will be successive numbers in the Fibonacci series. There is tremendous consistency within species regarding these numbers — in daisies, for example, the numbers [21, 34] are invariably found, while in sunflowers the numbers are [34, 55].[33] This asymmetric pattern is related to the geometry of each primordium, which, unlike the pans in the Palazetto, subtend different angles at each of their four vertices. A clue to the appearance of the Fibonacci series may lie in the universal appearance of a third logarithmic spiral — more tightly wound and not visually apparent — formed by a continuous curve connecting each primordia in order of appearance.[34] These lines are suggestive because of the geometrically self-replicating nature of the equiangular spiral and the connections between its mathematics and the Fibonacci series in geometry. It is possible, therefore, that the generative spiral is the shape based on capitular growth algorithms, whereas the parastichial spirals are simply the resultant manifestations of the shape's tendency to self-replicate.

We have, then, three instantiations of mathe-




7. Coneflower capitulum and diagram showing the manifestation of the Fibonacci series in flowering plants. (Photo by the author, diagram by the author after Alan Turing, *Morphogenesis*.)

matical logic in the physical world, each of which can be seen as the formal manifestation of simple — although complexly interactive — procedures of growth and construction. The nautilus's reliance on a one-dimensional "program" for sequential growth produces a single equiangular spiral to which each cell relates directly. The sunflower and the Palazetto each rely on multidimensional programs, and therefore the spirals manifest in each are the result of interactions between individual cells or primordia. In all three cases, the algorithms involved produce shapes that are similarly proportioned although differently scaled, in sequence and in relationships that are regular and gnomonic. In all three cases, the resultant forms are precise "diagrams" of the forces of assembly involved, records of the constructive logics inherent in the assemblies' codings, whether organic or architectural.

It is here that one turns back to the Palazetto to find the "parastichial order" of Nervi's system and thus, perhaps, to confirm a formal link between the lamella system and the organic world. And, of course, we are profoundly disappointed when we realize that the roof's order is the very ordinary [108, 108] — not only a pair of non-Fibonacci numbers, but also not even a sequential pair. In fact, the examination of both patterns reveals that, although the sunflower maintains a dynamic inequality in its rotational pattern, based on two different convergence angles in each primordia, the Palazetto demonstrates both local and rotational symmetry, with each primordia demonstrating the same convergence angle in either direction.

And yet, this is precisely what we should have expected, for Nervi's roofs share only limited algorithmical similarities with the processes of phyllotaxis. The roof of the Palazetto was constructed of ferro-cement pans assembled in their final shapes, whereas primordia emerge and grow within the fabric of the capitulum. Nervi's process relied on molds and jigs and the economies gained from reusing these numerous times. The roof pans therefore are consistently sized within each circumferential "family," whereas the primordia are sized differently depending on their time of emergence. Essentially, the algorithms of Nervi ensure maximum similarity in size and shape over the surface of the dome, whereas those of the sunflower ensure maximum density for dynamically changing elements. Rotational asymmetry appears in the sunflower as a signature of constant, incremental additions, and the symmetry of the Palazetto indicates its regularity and simultaneous configuration. With no change over time and no dynamic elements, we should therefore expect that the Fibonacci series, characteristic signature of growth, should in fact be absent from the concrete roof.

There is, too, a fundamental difference between the two sets of patterns in that the generative logarithmic spiral in phyllotaxis (the tightly wound connection between each primordia) is absent in the roofs of Nervi — or, rather, it and its generative tendency are replaced by a series of concentric circles. These reflect the substitution of constant growth in sunflowers by the system of simultaneous fabrication and parallel assembly in the Palazetto. The Palazetto roof can thus be considered as a capitulum formed entirely by constructive processes, assembled rather than grown, static rather than dynamic. There is no root justification for asymmetry; in fact, there is an economic requirement for constant sizes and angles throughout the construction. The crucial parallels between the two systems therefore lie not in their precise geometric correspondence, but rather in their reliance on simple algorithms and their resulting manifestations of geometrical figures that are signatures of staged, modular assembly. The subtle yet critical difference of asymmetry and the parallel appearance of the Fibonacci series in the organic model are evidence that they are formed by different processes. The relationships between the patterns in these roofs and those of spiral phyllotaxis are, therefore, neither perfect nor merely coincidental. They are instead examples of similar underlying mathematical principles instantiated under different conditions, with results that suggest continuity between the organic and the architectural while discretely clarifying where these processes logically differ.

That Thompson should have so underappreciated the subject of phyllotaxis is thus doubly unfortunate, for it is precisely at the level of growth and assembly that the distinction between these examples is most evidently expressed. Nervi's roofs demonstrate a constructional efficiency based in the mechanical mass-production of static elements, while sunflowers show us an equally striking example of organic efficiency based in continual growth. That there is great beauty to be found in each example is hardly surprising, given the elegance of the mathematics involved. However, what seems notable here is that each system contains a legible expression of its own constitution, a systemic and thorough marking of its own generative logic. It is striking that the intuitive sense in each case can be so profoundly touched — Nervi's roofs "appear" structurally correct, and the sunflower "appears" to be a natural outgrowth of its organic processes. In each case, the form of the material is a direct, easily apprehended manifestation of the processes at work. They are thus the "warp and woof of mechanism and teleology" made corporeal as signatures of the modes of making, diagrams indeed of forces at once beyond our comprehension yet within our understanding.

Perhaps the most intriguing suggestion of this parallel is that the most profound similarities between the architectural and the organic take place precisely in the realm of causation — that is, in the orchestration of motive and final cause, the "how" and the "why" of the phenomena involved. Nervi described these as the "essential problems" of architecture — the structural (final), the constructional (motive), and the architectural (expressive). That Nervi included this third element, subjective and slippery though it might be, was indicative of a humanist spirit that pervades his works of technical rigor. The visual joy offered by these roofs, Nervi wrote, was evidence of "the mysterious connection between the laws of physics and our esthetic sensibility."[35] In much the same way, Thompson noted that "whatsoever is beautiful and regular is also found to be most useful and elegant."[36] Here we find the ideal of organic beauty based not in composition, but rather in the humble revelations of processes that conduct and govern the worlds of the natural and the constructed.

Contemporary developments in formal generation and construction suggest parallels to the Palazetto. Nervi's constructions were designed, of course, without recourse to the array of curve generators allowed by CAD systems, and likewise the components of his formwork were laid up by hand, not manufactured by CNC-controlled devices. These contemporary tools have resulted in three basic approaches to the integration of form, structure, and assembly — one that seeks dramatic spatial potential from digitally derived constructions, one that explored the possibilities of CAD-CAM to allow variation in constructable form, and one that is interested in the adaptation of biomorphic elements as structural or cladding systems. Typified by Greg Lynn, Frank Gehry, and Santiago Calatrava, respectively, each of these stances suggests the organic at some level — either literally in the case of Calatrava's bone-like structural experiments, metaphorically in the case of Lynn's "biomorphic" architecture, or analogically in terms of Gehry's links between production, assembly, and morphology. In each case, however, it is apparent that the freedom allowed by digital methods removes the object from a sense of intuitive comprehension regarding the resultant form's performance or assembly process. Indeed, the forced reliance on "simple" geometry in Nervi's case — that is, geometry that could be manually constructed at both the drafting table and the job site — resulted in an integrated pattern that expressed both structure and construction. These qualities are occasionally absent in the impressive but often statically "incorrect" work of Calatrava or the seductive skins of Lynn and Gehry. This is not to dismiss the technical achievement of these works, only to suggest that their visual interest lies in a different sphere than that of Nervi's roofs. Whereas the former appeal through their denial or subversion of actual static form, the latter connect us with universal principles of statics and material. Like the physical forces that shape the examples in Thompson's study, the enforced simplicity of algorithm and pattern in the roof of the Palazetto fulfilled Nervi's threefold prescription for structural architecture: efficiency in structure and construction, and an expressive formal resolution of these. Nervi's equiangular roofs are diagrams of forces that show us, through simply conceived geometry that is nevertheless complex in appearance, patterns of performance and assembly in ways that express the interweaving of Thompson's "warp and woof," mechanism and teleology, making and working, and assembly and performance.

**Notes**

1. D'Arcy Thompson, *On Growth and Form* (Cambridge: University Press, 1917), p. 11.
2. "[Thompson's] hybrid theory of Pythagoras and Newton argues that physical forces shape organisms directly (with 'internal' and genetic forces responsible only for producing raw material, admittedly in gradients and programmed timings, for construction under principles of physics) — and that the ideal geometries beloved by classical Athens pervade organic forms because natural law favors such simplicity as an optimal representation of forces." Steven Jay Gould, foreword to the Cambridge University Press Canto edition of *On Growth and Form*, 1992.
3. Thompson was not only an accomplished anatomist but also a recognized classicist, mathematician, and lecturer. See Ruth D'Arcy Thompson, *D'Arcy Wentworth Thompson: The Scholar-Naturalist, 1860–1948* (London: Oxford University Press, 1958).
4. Most notably, even the second edition of *On Growth and Form* makes virtually no mention of genetics.
5. This point is made most clearly by John Tyler Bonner's "Editor's Introduction" to the Cambridge University Press Canto reprint of the abridged edition of *On Growth and Form*.
6. "I would venture to suggest that Aristotle inclined to slur over the physical and lean the more to the final cause, for this simple reason (whatever other reasons may be), that he was a better biologist than physicist." D'Arcy Thompson, "Aristotle the Naturalist" in D'Arcy Thompson, *Science and the Classics* (London: Oxford University Press, 1940), p. 70.
7. D'Arcy Thompson, *On Growth and Form*, p. 6.
8. Vagn Lundsgaard Hansen, *Geometry in Nature* (Wellesley, MA: A. K. Peters, 1993), p. 5.
9. William Allen Whitworth, B.A. "The Equiangular Spiral, Its Chief Properties Proved Geometrically," *The Oxford, Cambridge, and Dublin Messenger of Mathematics* 1 (1862): 5.
10. Robert Dixon, "The Mathematics and Computer Graphics of Spirals in Plants," *Leonardo* 16/2 (1983): 86–90. See also the seductively titled David Ward, The *Penguin Dictionary of Curious and Interesting Geometry* (London: Penguin Books, 1991), pp. 67–68.
11. "In short it is obvious that the *form* of an organism is determined by its rate of *growth* in various directions . . . organic form itself is found, mathematically speaking, to be a *function of time*." Thompson, *On Growth and Form*, p. 76.
12. Peter S. Stevens, *Patterns in Nature* (USA: Atlantic-Little, Brown, 1974), pp. 81–93. The classic study of spiral formation in nature remains Theodore Andrea Cook's *The Curves of Life* (London: Constable & Co., 1914, rep. New York: Dover Publications, 1979).
13. Werner Blaser, *After Mies: Mies van der Rohe — Teaching and Principles* (New York: Van Nostrand Rheinhold, 1977), p. 286, and Kenneth Frampton, *Studies in Tectonic Culture* (Cambridge, MA: The MIT Press, 1995), p. 216.
14. Myron Goldsmith, "The Effects of Scale," in *Myron Goldsmith: Buildings and Concepts* (New York: Rizzoli, 1987), pp. 9–10.
15. David Brownlee and David De Long, *Louis I. Kahn: In the Realm of Architecture* (New York: Rizzoli, 1991), pp. 60–61.
16. Frei Otto, *Form and Structure* (Boulder: Westview Press, 1976), p. 22.

17. Kenneth Frampton, "The Legacy of Alvar Aalto: Evolution and Influence," in *Alvar Aalto: Between Humanism and Materialism* (New York: Museum of Modern Art, 1998), p. 126. See also Kenneth Frampton, *Studies in Tectonic Culture* (Cambridge, MA: The MIT Press, 1995), p. 253, on quotations by Utzon of Thompson's illustrations. Additional recent mentions of Thompson in relation to architecture include Alexander Zannos, *Form and Structure in Architecture* (New York: Van Nostrand Rheinhold, 1987); Christopher Williams, *Origins of Form* (Stamford, CT: Architectural Book Publishing Company, 1981); and the proceedings of the 2001 ACSA East Central Regional Meeting, entitled "On Growth and Form: The Engineering of Nature," at which an early version of this paper was presented.

18. Pier Luigi Nervi, "The Importance of Construction Techniques," *Student Publications of the School of Design* 6/1 (Raleigh, NC: North Carolina State College, 1956): 12.

19. Pier Luigi Nervi. "Some Considerations About Structural Architecture," *Student Publications of the School of Design* 11/2 (Raleigh, NC: North Carolina State College, 1963): 43.

20. Pier Luigi Nervi, "On Relations Between Construction Processes and Architecture," *Student Publications of the School of Design, North Carolina State College* 6/2 (Raleigh, NC: North Carolina State College, 1956): 2.

21. G. R. Kiewitt, "The New Look of Lamella Roofs," in Robert E. Fischer, ed., *Architectural Engineering: New Structures* (New York: McGraw-Hill/Architectural Record, 1964), pp. 20–25. See also Mario Salvadori, *Structure in Architecture: The Building of Buildings*, 2nd ed. (Englewood Cliffs: Prentice-Hall, 1975), p. 214.

22. Pier Luigi Nervi, "The Importance of Construction Techniques," *Student Publications of the School of Design* 6/1 (Raleigh, NC: North Carolina State College, 1956): 7.

23. "Pier Luigi Nervi," *Architectural Forum* 99 (Nov. 1953): 140.

24. Pier Luigi Nervi, *Aesthetics and Technology in Building* (Cambridge, MA: Harvard University Press, 1965), p. 31.

25. Pier Luigi Nervi, "Concrete and Structural Form," *The Structural Engineer: The Journal of the Institution of Structural Engineers* 34/5 (1956): 158–161.

26. Pier Luigi Nervi, "Precast Concrete Offers New Possibilities for Design of Shell Structures," *Journal of the American Concrete Institute* (Feb. 1953): 543–546.

27. "Nervi's Olympic Dome," *Architectural Forum* 103/3 (March 1958): 84.

28. Ibid.

29. Photos of workers assembling pans for the third hall of the Turin exhibition hall show a steel jig being moved from the casting floor. Nervi, *Aesthetics and Technology in Building*, p. 130.

30. In the two limiting cases, the angle of incidence ($\theta$) could be 90 degrees, in which case the roof would be configured as a simple radial arch scheme, or 180 degrees, which would suggest a monolithic concrete shell.

31. This abridgement is explained by the editor John Tyler Bonner as having "contributed no new information to this old subject." However, the tone and Thompson's explanation — that the spirals are essentially optical illusions — contradict the generous character of his other chapters. D'Arcy Thompson, *On Growth and Form*, p. xx.

32. The following explanation is largely adapted from Roger V. Jean, *Phyllotaxis: A Systemic Study in Plant Morphogenesis* (Cambridge: Cambridge University Press, 1994). See also P. T. Saunders's introduction to Alan M. Turing, *Morphogenesis* (Amsterdam: Elsevier Science Publishers, 1992), pp. IX–XXIV. I am greatly indebted to Dr. Jean for his generous review of an early version of this paper and his clarification of the process described.

33. Przemyslaw Prusinkiewicz and Aristid Lindenmayer, *The Algorithmic Beauty of Plants* (New York: Springer-Verlag, 1990), pp. 99–107.

34. Turing, *Morphogenesis,* p. IX.

35. Nervi, "The Importance of Construction Techniques," p. 12.

36. Thompson, *On Growth and Form*, p. 327.